RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. FIRST SEMESTER (July – December), 2011

Mid-Semester Examination, September, 2011

Date : 12/09/2011

PHYSICS (Honours)

Time : 11 am – 1 pm

Paper : I

Full Marks : 50

Answer all questions

1.a) Let $\vec{a_1} = (-1, 1, 1)$, $\vec{a_2} = (1, -1, 1)$, $\vec{a_3} = (1, 1, -1)$. Show that $\{\vec{a_1}, \vec{a_2}, \vec{a_3}\}$ are linearly independent. Hence, obtain a set $\{\vec{b_1}, \vec{b_2}, \vec{b_3}\}$ reciprocal to this set. 4

b) Prove that a neccessary and sufficient condition that $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \times \vec{C} = 0$, is $(\vec{A} \times \vec{C}) \times \vec{B} = 0$. 3

c) Let $\vec{r}(t)$ be a vector function of t. Show that if

i) $\vec{r}(t) \times \frac{d\vec{r}}{dt} = \vec{0}$, then $\vec{r}(t)$ has a fixed direction.

ii) $\vec{r}(t) \cdot \frac{d\vec{r}}{dt} = 0$, then $\vec{r}(t)$ has a fixed magnitude. 3

OR

1.a) Distinguish between a polar and an axial vector, giving an example of each. 2

b) Consider two sets of vectors $\{\vec{a_1}, \vec{a_2}, \vec{a_3}\}$ and $\{\vec{b_1}, \vec{b_2}, \vec{b_3}\}$ such that $\vec{a_i} \cdot \vec{b_j} = \delta_{ij}$. { i, j = 1,2,3 }

i) Show that the vectors of each set are non coplanar.

ii) Hence express the $\{b_i\}$ in terms of the set $\{a_i\}$. 5

c) Prove from first principle that $\frac{d}{dt}\{ \vec{A}(t) \cdot \vec{B}(t) \} = \frac{d\vec{A}}{dt} \cdot \vec{B}(t) + \vec{A}(t) \cdot \frac{d\vec{B}}{dt}$. 3

2.a) Find a general solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$. 3

b) Find two solutions of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ assuming the solution of the form $y = \Sigma_{n=o}^{\infty} a_n x^n$. Calculate the Wronøskian of the solutions at x = 0 and comment on the result. 5

c) Assuming $(1 - 2xh + h^2)^{-\frac{1}{2}} = \Sigma_{n=0}^{\infty} h^n P_n(x)$, find the value of $P_n(1)$. 2

3.a) Expand $f(x) = x \quad 0 < x < \pi$, in Fourier series. 3

b) Show that Fourier transform of the Gaussian function is a Gaussian function. 3

c) Show that $\delta(-x) = \delta(x)$. 4

4.a) State Fermat's principle. Using this principle establish that all rays passing through the focus of a parabolic mirror are rendered parallel to the axis after reflection. 1+2

b) Name and define the cardinal points of a thick lens. 3

c) Find the cardinal points of a transparent sphere of a material having $\mu = 1.5$ and radius 2 cm when placed in air. 4

5.a) What do you mean by chromatic aberration ? 2

b) What is an achromatic optical system ? 2

c) Deduce the condition of achromatism of two lenses separated by a distance . 3

d) An object is placed at a distance 30 cm from a convex lens. The violet part of the image is at a distance 50 cm from the lens. If r.i. of the material of the lens for violet light is 1.64, determine the r.i. for the red light. Assume the lateral chromatic error is 4.2 cm. 3